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ABSTRACT
The paper deals with the detuning optimization of a mistuned bladed-disk in presence of geometrical nonlinearities. A full data basis is constructed by using a finite element model of a bladed-disk with cyclic order 12, which allows all the possible detuning configurations to be computed. It is then proposed to reformulate the combinatorial optimization problem in a probabilistic framework, using and adapting the recent Probabilistic Learning on Manifolds (PLoM) tool to the detuning context. The available full data basis is used in order to validate the proposed method.

Keywords: Detuning/Mistuning, Geometrical nonlinearities, Probabilistic learning on Manifold, PLoM, Uncertainty Quantification, Stochastic Optimization

INTRODUCTION
This research concerns the improvement of the vibratory performances of turbomachines by using a detuning optimization strategy that allows for reducing the

amplifications induced by the unavoidable random blade mistuning of bladed-disks. In a green aviation context for which fan blade design yields larger blades made up of lighter materials, nonlinear geometrical effects are taken into account. The detuning is described by using alternating patterns of several different sector types. A full data basis is constructed by using a finite element model of a bladed disk with cyclic order 12 [1, 2], which allows the random responses of all the possible detuning configurations to be identified [3]. Such a detuning optimization requires to solve an high-dimensional combinatorial optimization problem for which the cost function is evaluated from a nonlinear stochastic reduced computational model (High-Fidelity Computational Model (HFCM)), that has previously been constructed [3, 4]. In practical situations, only a small data training set, issued from the HFCM and which does not a priori include any optima, is available. The main idea is then to construct a continuous approximation of this cost function, based on the use of the Probabilistic Learning on Manifolds (PLoM) [5, 6], and that is used in the learning step. Several difficulties inherent to the definition of the cost function require to reformulate the definition of the optimum. The available full data basis is then used to validate the proposed methodology.

BACKGROUND

The bladed-disk is assumed to have n_w blades with two types of blades (labelled by integer 0 and 1). A detuning configuration ℓ is then parameterized by a vector $\mathbf{w}^{c,\ell} = (w_1^{c,\ell}, \ldots, w_{n_w}^{c,\ell})$, in which for $k \in \{1, \ldots, n_w\}$, w_k^c is equal to 0 or to 1. In the frame of the mistuning, the resonance of the most responding blade is defined by the $\mathbb{R}^+$-valued random variable A^ℓ, in which upperscript ℓ corresponds to the detuned configuration number ℓ and whose realization θ_k is denoted by $a^{\ell,k} = A^\ell(\theta_k)$. In order to get a robust scalar quantity for characterizing the random nonlinear dynamical behavior of the detuned structure, an estimate of the maximum extreme value statistics of random variable A^ℓ is constructed. The number of Monte Carlo numerical simulations is written as $n_{\rm sim} = \nu_r\,\nu_e$ (for $n_{\rm sim} = 500$, $\nu_e = 10$ and $\nu_r = 50$). For $\ell \in \{1, \ldots, \nu_e\}$, we then define the quantity $\underline{a}_M^\ell$, such that

$$\underline{a}_M^\ell = \frac{1}{\nu_r} \sum_{r=1}^{\nu_r} a_M^{\ell,r} \quad \text{with} \;\; a_M^{\ell,r} = \max_{k \in \{\nu_e(r-1)+1, \ldots, r\nu_e\}} a^{\ell,k} \; r \in \{1, \ldots, \nu_r\}\,.$$

It should be noted that such quantity of interest is neither issued from a mean value or from an extreme value but is defined as an average of a set of $\nu_r = 50$

maxima taken in a subset of $\nu_e = 10$ realizations. The observation of the detuned ℓ-configuration with mistuning is then defined as the amplification factor $q^{c,\ell}$ with respect to its tuned counterpart with pure mistuning, that is written as $q^{c,\ell} = \underline{a}_M^{\ell} / \underline{a}_M^{\mathrm{t}}$, in which superscript t is related to the tuned configuration. It is thus interpreted as an highly nonlinear function of $\mathbf{w}^{c,\ell}$ that is to say $q^{c,\ell} = f_{\mathrm{HFCM}}(\mathbf{w}^{c,\ell})$. An available data basis [3] is constructed using the finite element model of the blisk described in [1], yielding 352 detuning configurations that are restricted to the set $\mathcal{C}_c \subset \mathcal{N}_c$ of the $n_c = 216$ detuning configurations having a majority of blades with type 0.

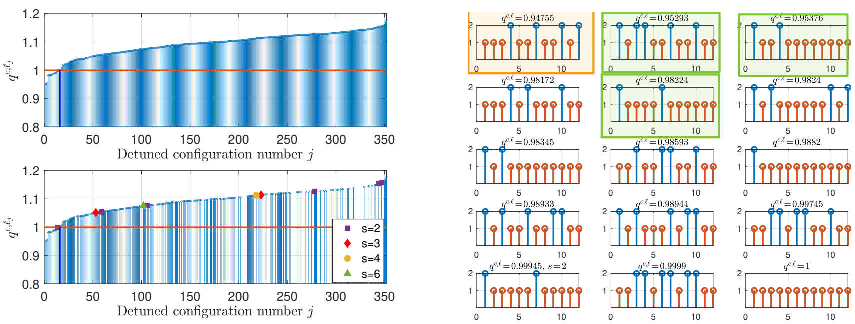

Figure 1: Quantity of interest according to the detuned configuration. Graph of function $j \mapsto q^{c,\ell_j}$ for the $n_c = 352$ possible detuned configurations (left upper figure) and for the $n_c = 216$ detuned configurations with a number of blades with type 2 less than or equal to 6 (left lower figure). Sub-cyclicity order s is also given. Characteristics of the improving detuned configurations: graph of $i \mapsto w_i^{c,\ell_j}$, $j \in \{1, \ldots, 15\}$ (right figures).

The detuning optimization consists in solving the combinatorial optimization problem such that optimum $\mathbf{w}_{\mathrm{un}}^{c,\mathrm{opt}}$ is defined by

$$\mathbf{w}_{\mathrm{un}}^{c,\mathrm{opt}} = \arg \min_{\mathbf{w}^c \in \mathcal{C}_c} \mathcal{J}_c(\mathbf{w}^c) \;\text{ with }\; \mathcal{J}_c(\mathbf{w}^c) = f_{\mathrm{HFCM}}(\mathbf{w}^c) \quad , \quad \mathbf{w}^c \in \mathcal{C}_c \,. \tag{1}$$

Such an optimization problem cannot in general be performed, the number of possible detuning configurations increasing exponentially with the number of blades. Let $\mathcal{D}_d \subset \mathcal{C}_c \subset \mathcal{N}_c$ be a N_d dimensional small training set with $N_d \ll n_c$ that does not a priori contains any improving detuning configurations and for which $\mathcal{J}_c(\mathbf{w}^c)$ is computed with HFCM for all $\mathbf{w}^c \in \mathcal{D}_d$. A new approximation of the optimization problem is formulated, yielding the optimum $\mathbf{w}^{c,\text{opt}}$ to be defined by

$$\mathbf{w}^{c,\text{opt}} = \arg \min_{\mathbf{w}^c \in \mathcal{C}_c} J_{\text{ar}}(\mathbf{w}^c) \text{ with } J_{\text{ar}}(\mathbf{w}^c) = \mathcal{E}\{Q|\mathbf{W} = \mathbf{w}^c\} \quad , \quad \mathbf{w}^c \in \mathcal{C}_c \, , \tag{2}$$

where $(Q, \mathbf{W})$ is a $\mathbb{R}^{1+n_w}$-valued random variable whose joint probability density function is constructed from the training data set using the modified multi-density Gaussian KDE [7] by adjusting a bandwidth parameter to get for each marginal density function of W_i a bi-modal probability density function centered around 0 and 1. The authors refer to [5–7] for the theory and the algorithm of the Probabilistic Learning on Manifolds (PLoM). This machine learning tool allows for generating a large number N_{ar} of learned realizations, denoted by $(q^k_{\text{ar}}, \mathbf{w}^k_{\text{ar}})$, $k \in \{1, \cdots, N_{\text{ar}}\}$, while preserving the concentration of the learned probability measure on the manifold defined by the graph $(q(\mathbf{w}), w \in \mathbb{R}^{n_w})$.

Due to the numerous local minima and weak contrast of highly nonlinear cost function $J_{\text{ar}}(\mathbf{w})$, the formulation of this optimization problem must be improved, leading us to reformulated the detuning optimization problem as follows. By fixing a parameter n_s defining the dimension of the set $\mathcal{W}^{\text{opt}}_{n_s} = \{\mathbf{w}^{c,\ell_1}, \ldots, \mathbf{w}^{c,\ell_{n_s}}\} \subset \mathcal{C}_c$ and by sorting $J_{\text{ar}}(\mathbf{w}^{c,\ell_1})$ for $j = 1, \ldots, n_s$, according to its first n_s increasing values, the optimal detuning configuration is then defined by

$$\mathbf{w}^{c,\ell_{\text{opt}}} = \arg \min_{\mathbf{w}^{c,\ell_j},\, j=1,\ldots,n_s} \{\mathcal{J}_c(\mathbf{w}^{c,\ell_1}), \ldots, \mathcal{J}_c(\mathbf{w}^{c,\ell_{n_s}})\}, \tag{3}$$

the existence of such an optimum being conditioned by the relation $\mathcal{J}_c(\mathbf{w}^{c,\ell_{\text{opt}}}) < \min_{\ell=1,\ldots,N_d} f_{\text{HFCM}}(\mathbf{w}^{c,\ell})$.

ANALYSIS

The proposed approach is applied using the available full data basis issued from the stochastic nonlinear computational model of a detuned bladed-disk structure with 12 blades and with 2 types of sectors. With such a data basis, there are $n_c = 216$ possible detuning configurations with a majority of blades with type 0, and the dimension N_d of subset $\mathcal{D}_d$ can vary from 1 to 202. The following table summarizes the main optimization results. It is seen that the improving detuning configurations represented in figure 1 and highlighted in green can be found as optimal solutions with a reasonable number N_d of training points. Note that such an algorithm converges to the optimal solution $\mathbf{w}_{\text{un}}^{c,\text{opt}}$ as defined in Eq. (2) and represented in orange in Figure 1, when all the $n_c = 216$ possible detuning configurations are included in the training set, which makes this formulation as a coherent one.

N_d	$j = 1,\ldots,6$	$\mathcal{J}_c(\mathbf{w}^{c,\ell_j}), \mathbf{w}^{c,\ell_j} \in \mathcal{W}_{n_s}^{\text{opt}}$						ℓ_{opt}	$q^{c,\ell_{\text{opt}}}$
50	q^{c,ℓ_j}	0.9995	0.9974	0.9834	1.0000	**0.9822**	1.0015	141	0.9822
75	q^{c,ℓ_j}	0.9974	0.9882	1.0000	1.0015	1.0067	**0.9529**	166	0.9529
100	q^{c,ℓ_j}	0.9974	1.0000	0.9995	**0.9538**	0.9822	0.9882	123	0.9538
202 (whole $\mathcal{D}_d$)	q^{c,ℓ_j}	1.0000	1.0015	0.9882	**0.9538**	1.0067	0.9822	123	0.9538
Reference	q^{c,ℓ_j}	**0.9476**	0.9529	0.9538	0.9822	0.9834	0.9824	104	0.9476

Table 1: Results as a function of the number N_d of training points in $\mathcal{D}_d$ - results obtained for $N_d = n_c = 216$ (reference).

CONCLUSION

In a detuning optimization context involving a large number of possible detuned configurations, we have proposed a reformulation in a probabilistic framework of the combinatorial optimization problem, which is adapted to a probabilistic machine learning tool in order to limit the number of evaluations of the cost function with the high-fidelity computational model. The methodology proposed has been validated for a 12-bladed-disk structure for which the exact optimal detuning configuration in presence of random mistuning has been previously identified. A good prediction of the optimum has been obtained with this method, that demonstrates the efficiency and the capability of the proposed methodology.

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